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ON SATELLITE TYPE ANTENNAS

by

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ABSTRACT

Using some previous study of the radiation of an antenna over a perfectly conducting sphere, various structures of antennas for space vehicles are discussed theoretically. The currents induced on the sphere are taken into account for the computation of radiation patterns.

ON SATELLITE TYPE ANTENNAS

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I. Introduction

C. H. Papas and R. W. King have shown[1], using the general reciprocity theorem, that for a linear antenna of length h over a perfectly conducting sphere of radius a (Figure 1), the following integral equation holds:

$$\int_0^{2\pi} E_{2\theta}\, I_{1S}(\theta)\, a d\theta = \int_a^{a+h} E_{2r}\, I_{1A}(r)\, dr \tag{1}$$

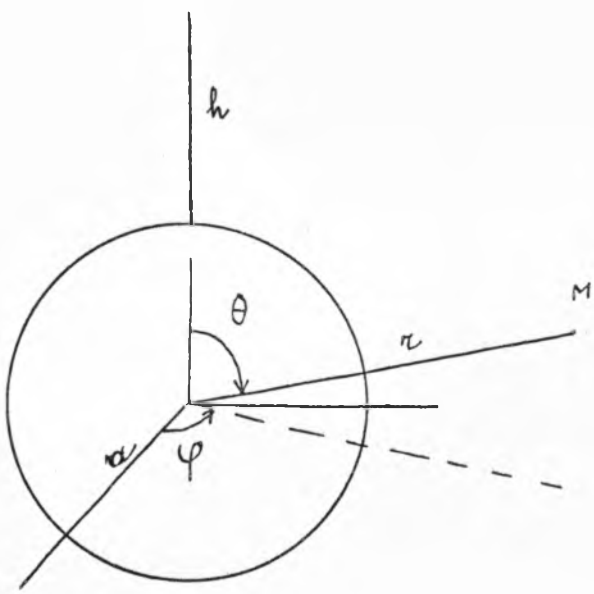


Figure 1.

where: $E_{2\theta}$ is a prescribed test field

$E_{2\theta} = U \sin\theta \cdot Pn(\cos\theta)$

E_{2r} is obtained from $E_{2\theta}$ by the solution of the boundary value problem

I_{1A} is the given antenna current

I_{1S} is the current on the sphere (which will be obtained by solving Eq. 1) .

II. Solution of the Integral Equation

As in Reference 1, we assume we have only TM modes, namely:

$$
\begin{aligned}
E_r &= \left(k^2 + \frac{\partial^2}{\partial r^2}\right) u \\
E_\theta &= \frac{1}{r} \frac{\partial^2}{\partial r\, \partial\theta} u \qquad (2) \\
H_\phi &= -\frac{ik}{r} \frac{\partial}{\partial\theta} u
\end{aligned}
$$

where u is a solution of the scalar wave equation

$$\nabla^2 \left(\frac{u}{r}\right) + k^2\left(\frac{u}{r}\right) = 0$$

The general solution for u is :

$$u = \sum_{m=0}^{\infty} A_m P_m(\cos\theta) \quad \rho_m(kr) \qquad (3)$$

where $\rho_m(x) = \sqrt{\frac{\pi x}{2}} \; H^{(2)}_{m+1/2}(x)$ is the weighted spherical function of the second kind.

We are looking for a solution: $I_{1S} = \sum_{n=0}^{\infty} B_n P_n(\cos\theta)$ (4)

If the antenna current is $I_{1A} = I_{max} \sin k(a-r)$, by identification we get from Eq. 1 ,

$$B_n = \frac{I_{max}}{2}\left[\frac{\rho_{n-1}(kd)}{\rho'_{n-1}(ka)} - \cos kh \frac{\rho_{n-1}(ka)}{\rho'_{n-1}(ka)} - \frac{\rho_{n+1}(kd)}{\rho'_{n+1}(ka)} + \cos kh \frac{\rho_{n+1}(ka)}{\rho'_{n+1}(ka)}\right] \tag{5}$$

III. The Case of Two Antipodal "In Phase" Antennas

By superposition the total current will be

$$I = \sum_{n=0}^{\infty} B_n \left[P_n(\cos\theta) - P_n(\cos(\pi-\theta))\right] = \sum_{\lambda=0}^{\infty} 2B_{2\lambda+1} P_{2\lambda+1}(\cos\theta) \tag{6}$$

all coefficients with even indices are zero.

IV. Explorer-Type Antenna

If we have two pairs of antipodal antennas, orthogonal to each other, and driven in phase quadrature, (Figure 2), we get by superposition:

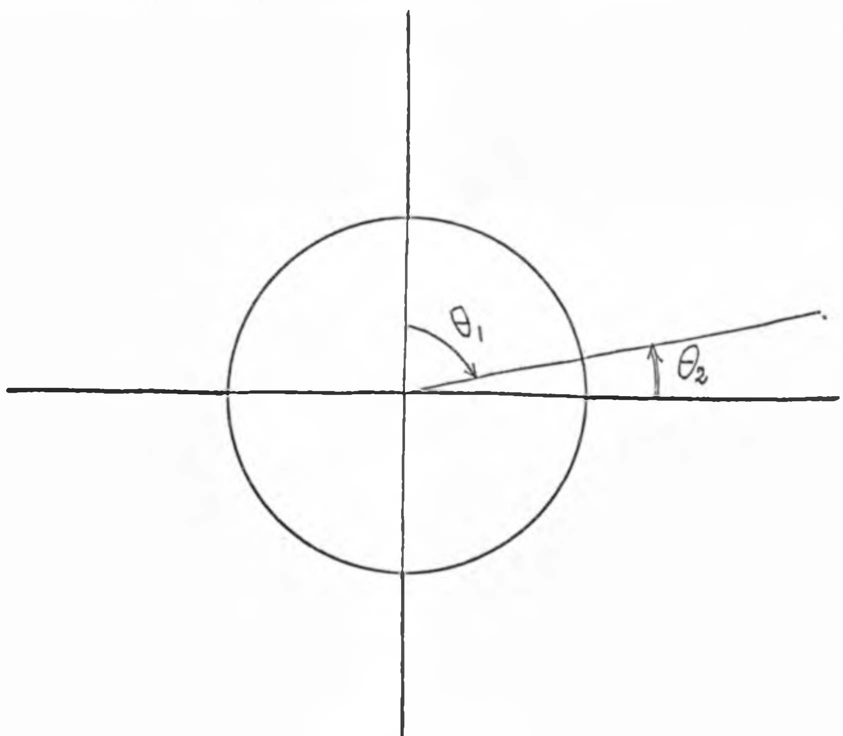


Figure 2.

$$I_{1S_{total}} = I_{max} \sum_{\lambda=0}^{\infty} \left[B_{2\lambda+1} P_{2\lambda+1}(\cos \Theta_1) + (B_{2\lambda+1} P_{2\lambda+1}(\cos \Theta_2))e^{i\frac{\pi}{2}} \right]$$

Θ_1 and Θ_2 are polar angles in Figure 2.

Coming back to the normal polar system in Figure 3, we get

$$I_{1S_{total}} = I_{max} \sum_{\lambda=0}^{\infty} B_{2\lambda+1} \left[P_{2\lambda+1}(\sin \Theta \cos \phi) + e^{\frac{i\pi}{2}} P_{2\lambda+1}(\sin \Theta \sin \phi) \right]. \tag{7}$$

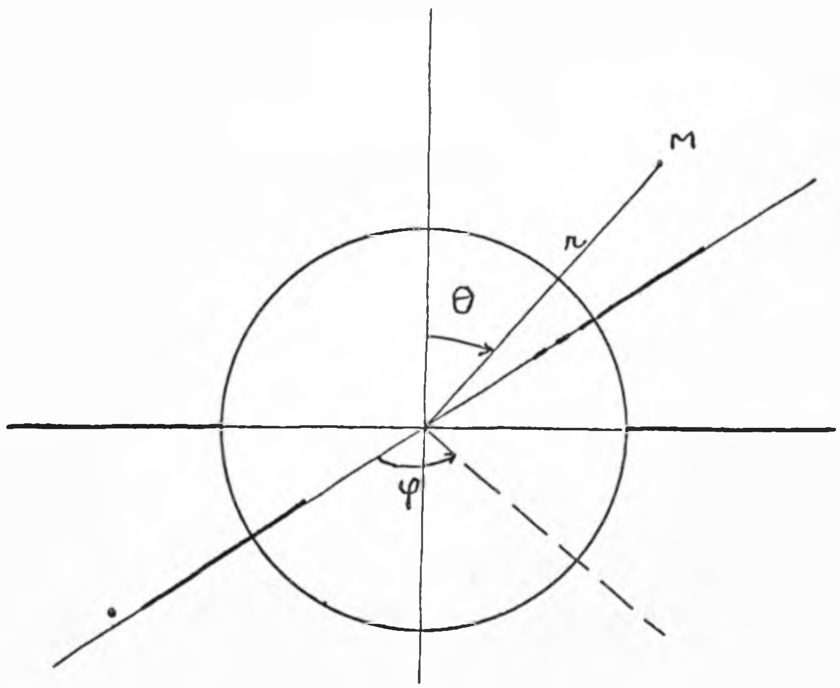


Figure 3.

V. Radiation Pattern

We have to superpose the radiation pattern of the antenna isolated in space and the field of the currents induced on the sphere. We had, for a pair of antipodal antennas,

$$u = \sum_{m=0}^{\infty} A_m P_m(\cos \theta_1) \rho_m(kr) \quad ,$$

in general,

$$H_{2\phi_1} = -\frac{ik}{r} \frac{\partial u}{\partial \theta} = -\frac{ik}{r} \sum_{m=0}^{\infty} A_m \frac{dP_m}{d\theta_1} (\cos \theta_1) \rho_m(kr) \quad .$$

But for $r = a$, $\quad H_{2\phi_1} = \dfrac{I_{1S}}{2\pi a \sin \theta_1} \quad .$

We know I_{1S} by Eq. (4). Hence an identification between coefficients gives us

$$H_{2\phi_1} = \frac{+ I_{max}}{2\pi r \sin \theta_1} \sum_{\lambda=0}^{\infty} \left\{ \frac{\rho_{2\lambda+2}(kr)}{\rho_{2\lambda+2}(ka)} P_{2\lambda+3}(\cos \theta_1) - P_{2\lambda+1}(\cos \theta_1) \; C \right\}$$

where $\displaystyle C_\lambda = \cos kh \frac{\rho_{2\lambda+2}(ka)}{\rho'_{2\lambda+2}(ka)} - \frac{\rho_{2\lambda+2}(kd)}{\rho'_{2\lambda+2}(ka)} - \cos kh \frac{\rho_o(ka)}{\rho'_o(ka)} + \frac{\rho_o(kd)}{\rho'_o(ka)} \quad .$

To get the complete radiation pattern in the case of the complete explorer antenna we have to add in quadrature the same expression with θ_1 replaced by θ_2 . (I_{max} replaced by $I_{max} \, e^{i\pi/2}$).

We have, of course

$$\cos \theta_1 = \sin \theta \cos \phi$$
$$\cos \theta_2 = \sin \theta \sin \phi \quad .$$

The actual radiation pattern is obtained by taking the asymptotic expansion for $\rho_{2\lambda+2}(kr)$

$$\rho_{2\lambda+2}(kr) \cong i^{2\lambda+3}\, e^{-ikr} \quad .$$

Hence we replace in the formulas $\dfrac{\rho_{2\lambda+2}(kr)}{r}$ by $(-1)^{\lambda+1}\, i$.

VI. The Case of the Tetrahedral Array

Let us now suppose that our four antennas pierce the unit sphere at points which are vertices of a regular tetrahedron. Positions of the four antennas are described in Figure 4, where two lie in the plane $y = 0$, two in the plane $x = 0$, and $\sin \psi = \sqrt{2/3}$. (See Fig. 6).

Superposing four times the solution obtained in Reference 1, we get for I_s the expression:

$$I_s = \sum_{n=0}^{\infty} B_n \quad \alpha\, P_n(\cos \theta_1) + \beta\, P_n(\cos \theta_2) + \gamma\, P_n(\cos \theta_3) + \delta\, P_n(\cos \theta_4)$$

$\alpha, \beta, \gamma, \delta$ being the phase factors.

$$B_n = -\frac{I_{max}}{2} \quad \frac{\rho_{n-1}(kd)}{\rho_{n-1}(ka)} - \cos kh\, \frac{\rho_{n-1}(ka)}{\rho'_{n-1}(ka)} - \frac{\rho_{n+1}(kd)}{\rho'_{n+1}(ka)} + \cos kh\, \frac{\rho_{n+1}(ka)}{\rho'_{n+1}(ka)}$$

and

$$\cos \theta_1 = \sin \theta \sin \psi \cos \phi - \cos \theta \cos \psi$$

$$\cos \theta_2 = -\sin \theta \sin \psi \cos \phi - \cos \theta \cos \psi$$

$$\cos \theta_3 = \sin \theta \sin \psi \sin \phi + \cos \theta \cos \psi$$

$$\cos \theta_4 = -\sin \theta \sin \psi \sin \phi + \cos \theta \cos \psi \quad .$$

To use completely the symmetries of the tetrahedron, the following combinations for $\alpha, \beta, \gamma, \delta$ may be used:

$(1, i, -1, -i)$ $\quad (1,1,1,1)$ $\quad (1,i,1,i)$ $\quad (1,i,i,1)$

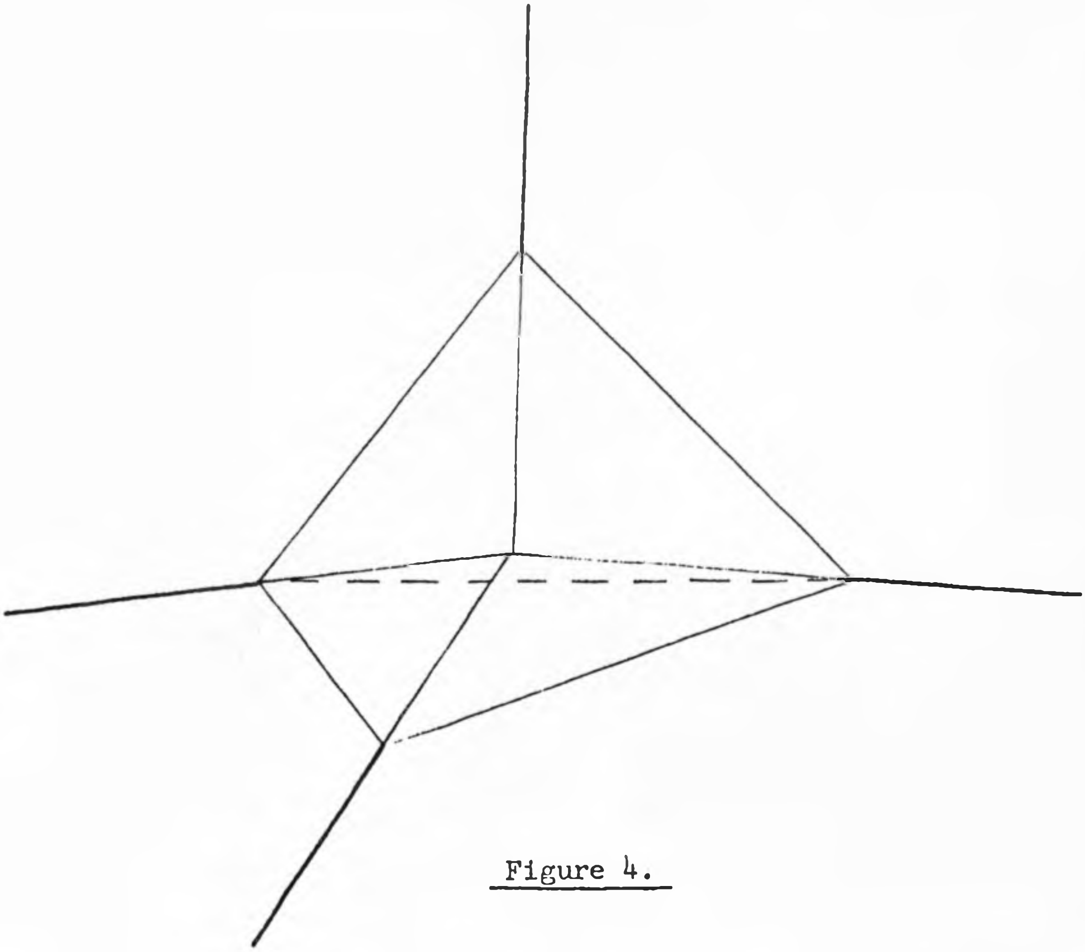


Figure 4.

To compute the field of one antenna, proceeding by identification we get:

$$H_{2\phi_1} = -\frac{ik}{r}\sum_{m=1}^{\infty} A_m \frac{d\,P_m}{d\theta_1}(\cos\theta_1)\,\rho_m(hr) \quad ;$$

with
$$A_{2\ +1} = -\frac{I_{max}}{4\pi\ ik}\cdot\frac{4\ \ +3}{(2\ +1)(2\ +2)}\cdot\frac{1}{\rho_{2\ +1}(ka)}\quad\frac{\rho_{2\ +1}(kd)}{\rho'_{2\ +1}(ka)} - \cos kh$$

$$\times\ \frac{\rho_{2\ +1}(ka)}{\rho'_{2\ +1}(ka)}\ ,$$

$$A_{2\ +2} = -\frac{I_{max}}{4\pi ik}\cdot\frac{4\ +5}{(2\ +2)(2\ +3)}\cdot\frac{1}{\rho_{2\ +2}(ka)}\quad\frac{\rho_{2\ +2}(kd)}{\rho'_{2\ +2}(ka)} - \cosh kh$$

$$\times\ \frac{\rho_{2\ +2}(ka)}{\rho'_{2\ +2}(ka)} - \frac{\rho_o(kd)}{\rho_o(ka)} + \cos kh\,\frac{\rho_o(ka)}{\rho'_o(ka)}\ .$$

To get the radiation pattern, we add geometrically the $H_{2\phi_i}$, $i = 1\cdots 4$

corresponding to the four antennas, and we get, replacing $\rho_m(hr)/r$ by $(i)^{m+1}$

$$H_\phi \cong \sum_{m=1}^{\infty} (i)^{m+1} A_m \left[\alpha \frac{d\, P_m(\cos \Theta_1)}{d\Theta_1} \cos x_1 + \beta \frac{d\, P_m(\cos \Theta_2)}{d\Theta_2} \cos x_2 \right.$$

$$\left. + \gamma \frac{d\, P_m(\cos \Theta_3)}{d\Theta_3} \cos x_3 + \delta \frac{d\, P_m(\cos \Theta_4)}{d\Theta_4} \cos x_4 \right]$$

with

$$\cos x_1 = - \frac{|\cos \Theta \sin \psi \cos \phi + \sin \Theta \cos \psi|}{\sqrt{\sin^2\psi \sin^2\phi + (\cos \Theta \sin \psi \cos \phi + \sin \Theta \cos \psi)^2}},$$

$$\cos x_2 = - \frac{|\cos \Theta \sin \psi \cos \phi - \sin \Theta \cos \psi|}{\sqrt{\sin^2\psi \sin^2\phi + (\cos \Theta \sin \psi \cos \phi - \sin \Theta \cos \psi)^2}},$$

$$\cos x_3 = \frac{|\cos \Theta \sin \psi \sin \phi - \sin \Theta \cos \psi|}{\sqrt{\sin^2\psi \cos^2\phi + (\cos \Theta \sin \psi \sin \phi - \sin \Theta \cos \psi)^2}},$$

$$\cos x_4 = \frac{|\cos \Theta \sin \psi \sin \phi + \sin \Theta \cos \psi|}{\sqrt{\sin^2\psi \cos^2\phi + (\cos \Theta \sin \psi \sin \phi + \sin \Theta \cos \psi)^2}},$$

H_Θ is obtained by putting $\sin x_i$ instead of $\cos x_i$ (Figure 5.)

VII. Complete Radiation Pattern

It is now necessary to add to the field of the current distribution on the sphere, the field of the given linear antennas. For a single antenna we get, from the well-known formula:[2]

$$H_{\phi_2} = -\frac{ik}{4\pi}\sin\theta\int_a^{a+h} e^{-k\xi\cos\theta}\, I_{max}\sin k(d-\xi)\,d\xi$$

$$\cong -\frac{i\, I_{max}}{8\pi}\sin\theta\, F(\theta)$$

where

$$F(\theta) = \frac{e^{-ikd\cos\theta} - e^{-ika(1+\cos\theta)+ikd}}{1+\cos\theta}$$

$$+ \frac{e^{ikd\cos\theta} - e^{ika(1-\cos\theta)-ikd}}{1-\cos\theta}$$

In the particular case of a quarter-wave antenna:

$$k(a-d) = k\ell = \pi/2 .$$

For any type of structure, we have to add geometrically the fields corresponding to several similar antennas.

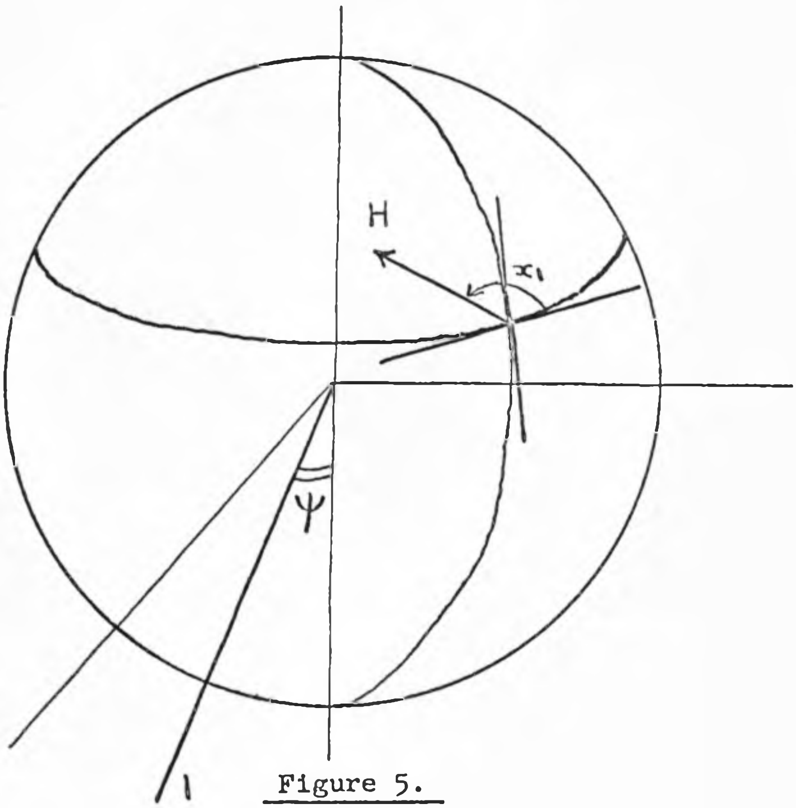


Figure 5.

Computational Results

Curves showing the radiation patterns have been drawn in several planes (See Figs. 6 and 7).

The case of four antennas in the same plane is described in Fig. 6.

Case I : $\theta = \frac{\pi}{2}$

Case II: $\phi = 0$

Case III: $\phi = \frac{\pi}{4}$.

For the tetrahedral array, results are shown on Fig. 7.

Case IV : $\theta = \frac{\pi}{2}$

Case V : $\phi = 0$

Case VI : $\phi = \frac{\pi}{4}$.

(We used the combination $(1, i, i^2, i^3)$ for better symmetry).

Case I (Figs. 8 and 9)

Only the part $0 \leq \phi \leq \frac{\pi}{4}$ has been represented. The magnitude of E_ϕ is plotted for

$$\left.\begin{array}{l} ka = \pi/4 \\ kd = 3/8\,\pi \end{array}\right\} \text{ on Figure 8}$$

$$\left.\begin{array}{l} ka = \pi/8 \\ kd = \pi/4 \end{array}\right\} \text{ on Figure 9 .}$$

The curves are symmetric with respect to the axis and the bisectors.

Case II (Figs. 10 and 11)

The magnitude of E_ϕ is plotted for $0 \leq \theta \leq \frac{\pi}{2}$

Figure 10 corresponds to Figure 8

Figure 11 corresponds to Figure 9

The curves are symmetric with respect to the axes.

Case IV (Fig. 12)

The magnitude of E_ϕ is plotted for $0 \leq \phi \leq 2\pi$. The curve is symmetric with respect to the x axis. (Scale in dbs).

$$Ka = \frac{\pi}{4} \qquad Kd = \frac{3\pi}{8} \quad .$$

Case V (Fig. 13)

The magnitude of E_ϕ is plotted for $0 \leq \theta \leq \pi$

The curve is symmetric with respect to the x-axis. (Scale in dbs).

$$Ka = \frac{\pi}{4} \qquad Kd = \frac{3\pi}{8} \quad .$$

The authors are greatly indebted to Dr. C. H. Papas for suggesting this study.

REFERENCES

1. C. H. Papas, R. W. King: "Surface Currents on a Conducting Sphere Excited by a Dipole", JAP 19, 1948, p.808.

2. J. Kraus, Antennas, McGraw-Hill.

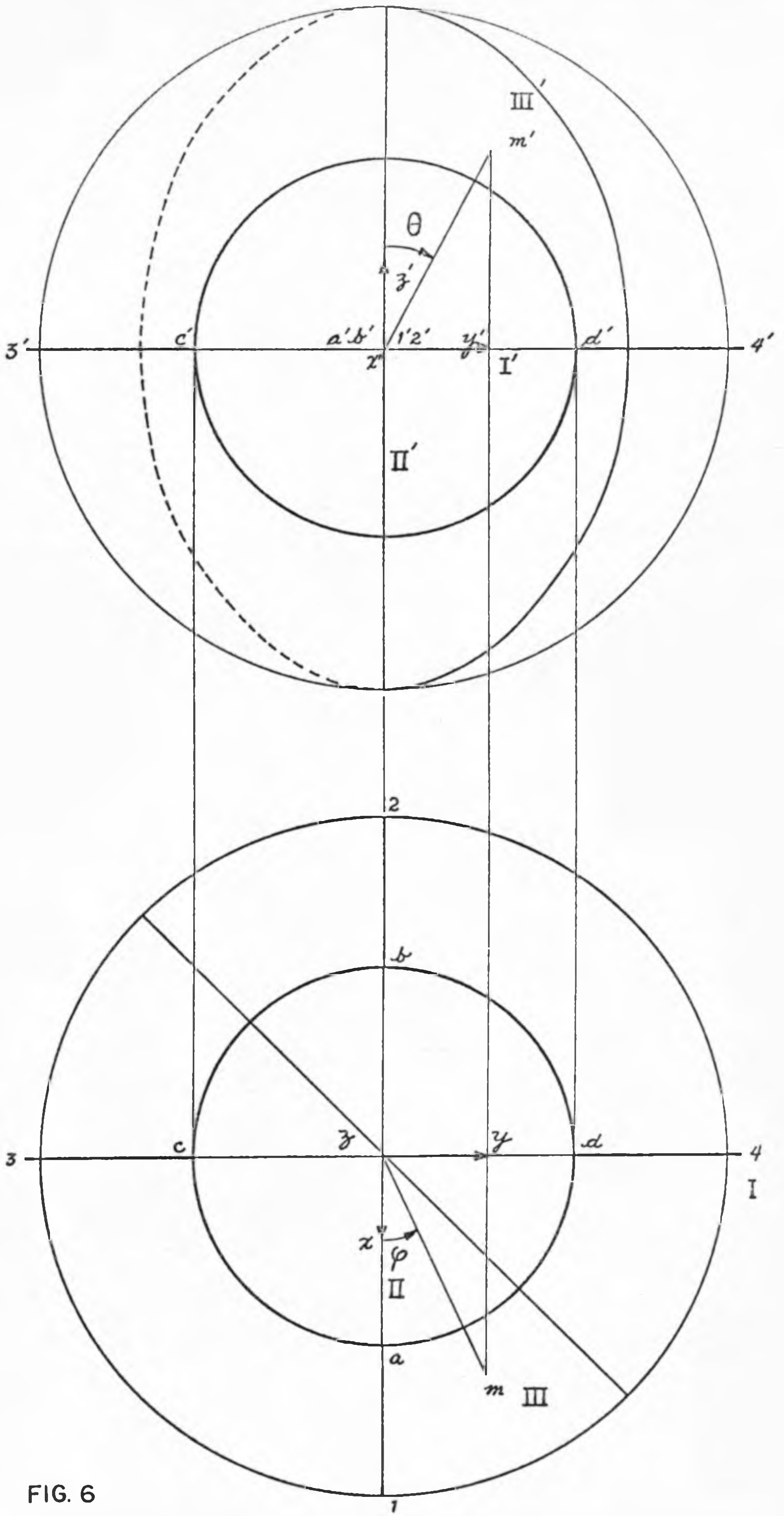


FIG. 6

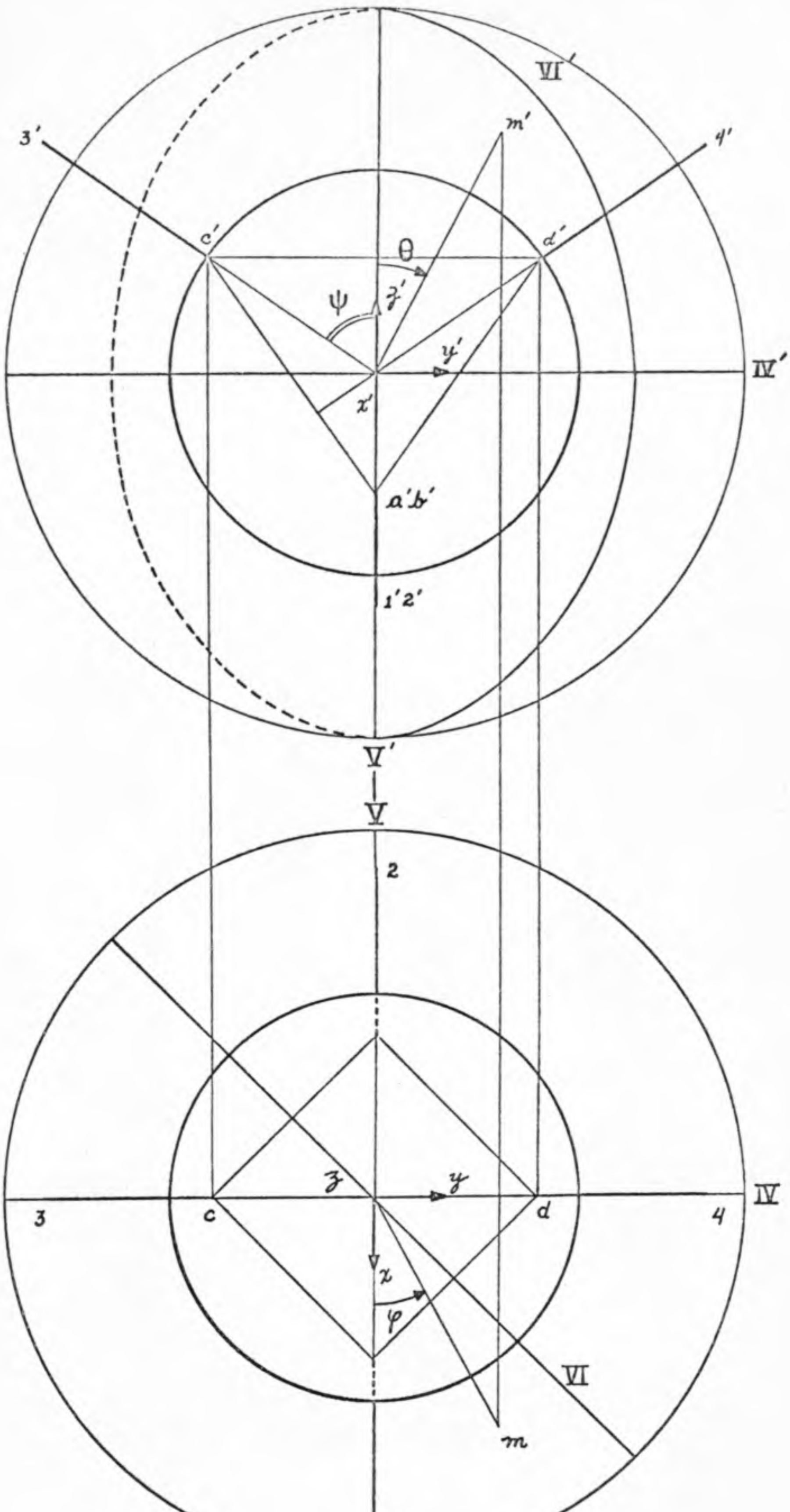


FIG. 7

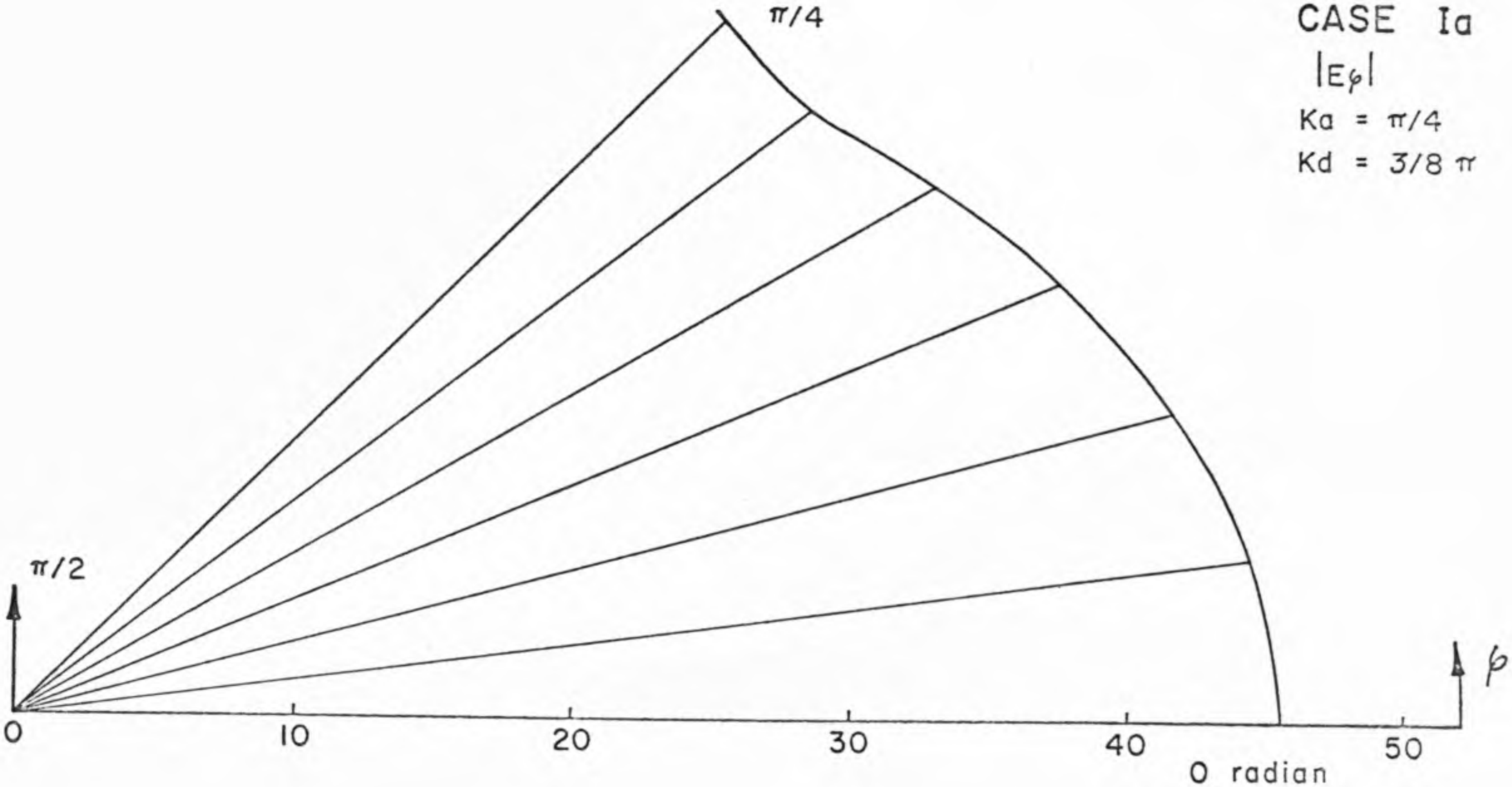


FIG. 8

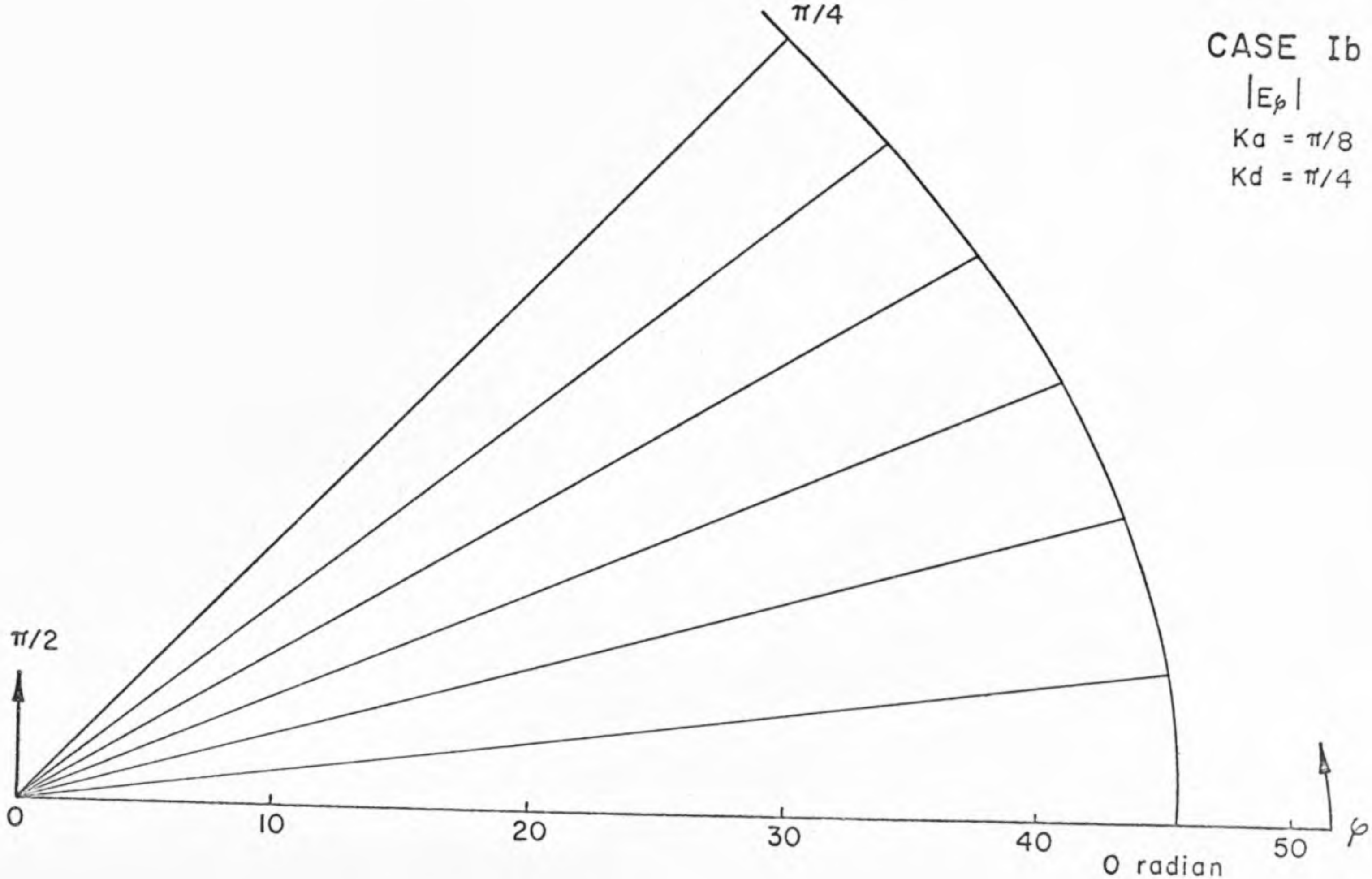


FIG. 9

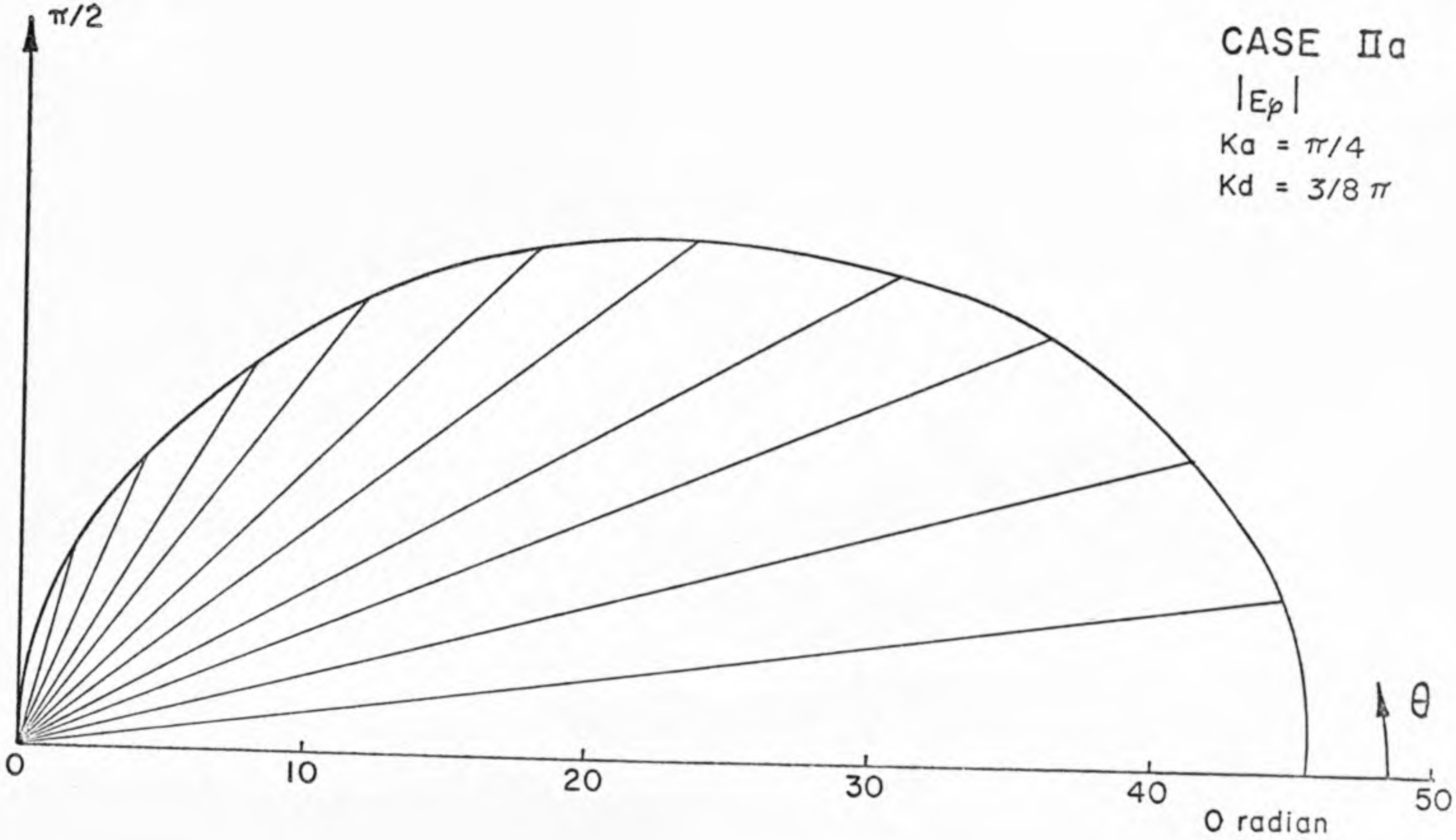


FIG. 10

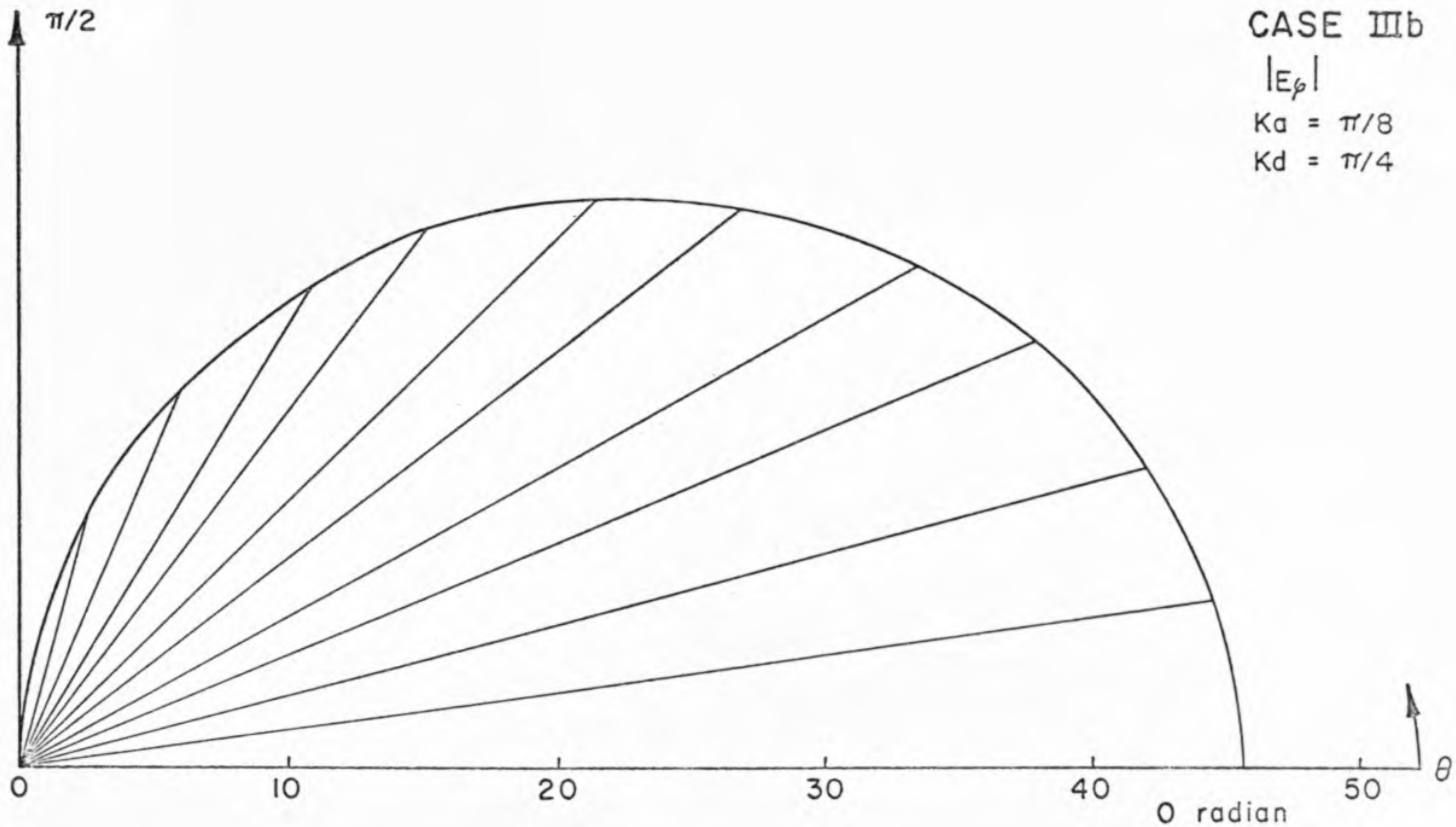


FIG. 11

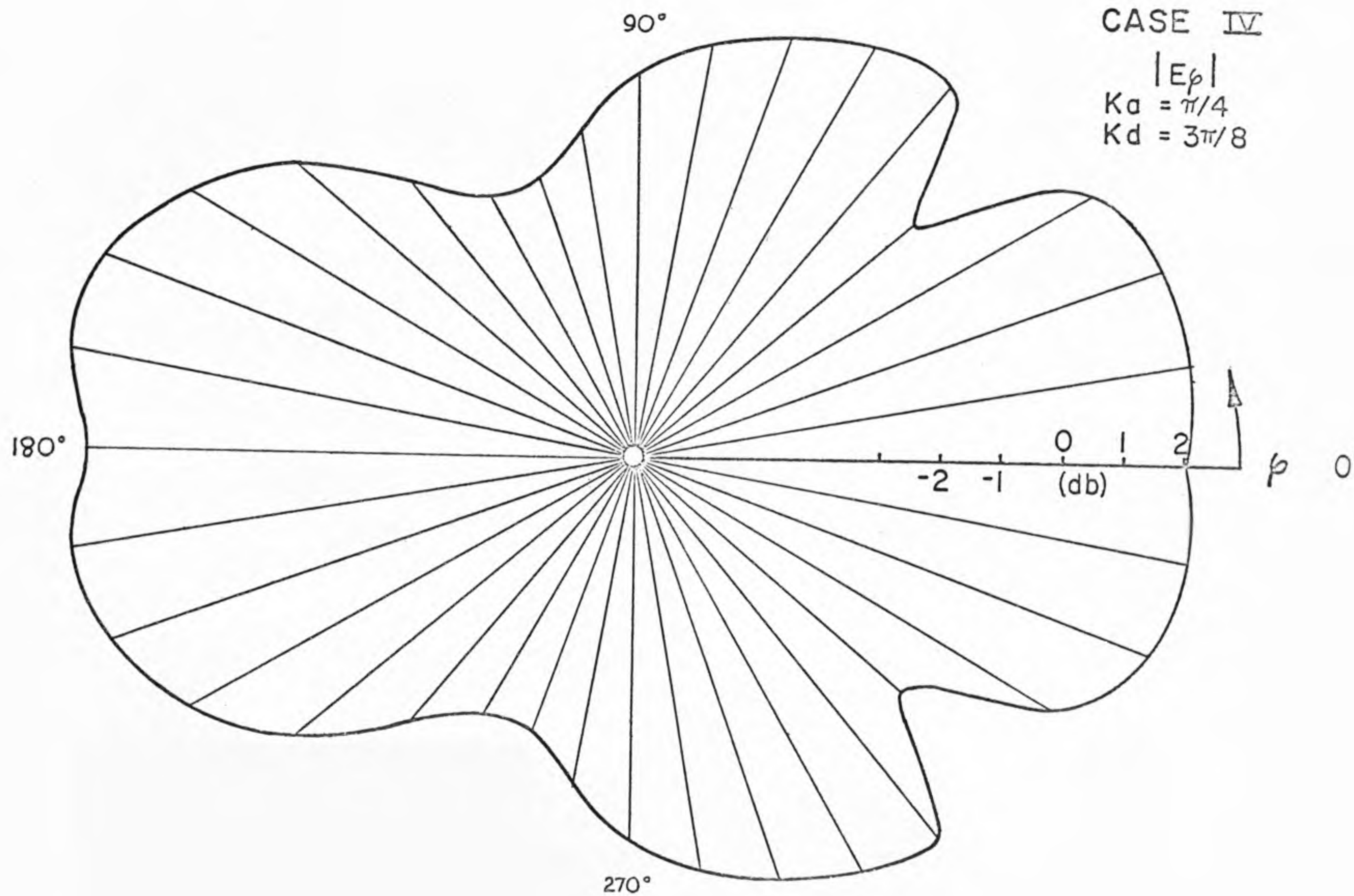
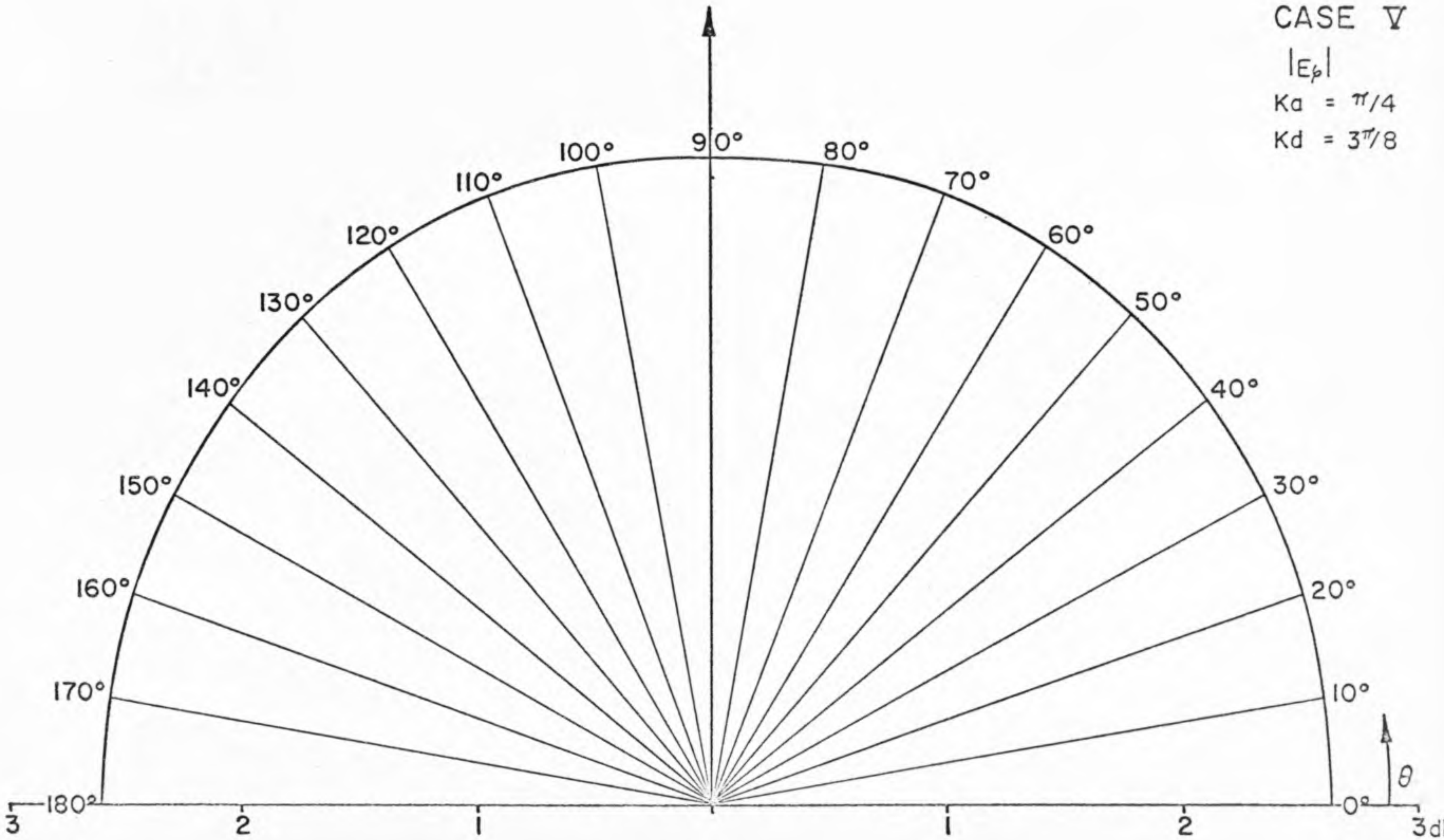


FIG. 12



CASE V
$|E_\phi|$
$Ka = \pi/4$
$Kd = 3\pi/8$
90°
100°
80°
110°
70°
120°
60°
130°
50°
140°
40°
150°
30°
160°
20°
170°
10°
180°
0°
θ
3
2
1
1
2
3db

FIG. 13